Intermediate Macroeconomics

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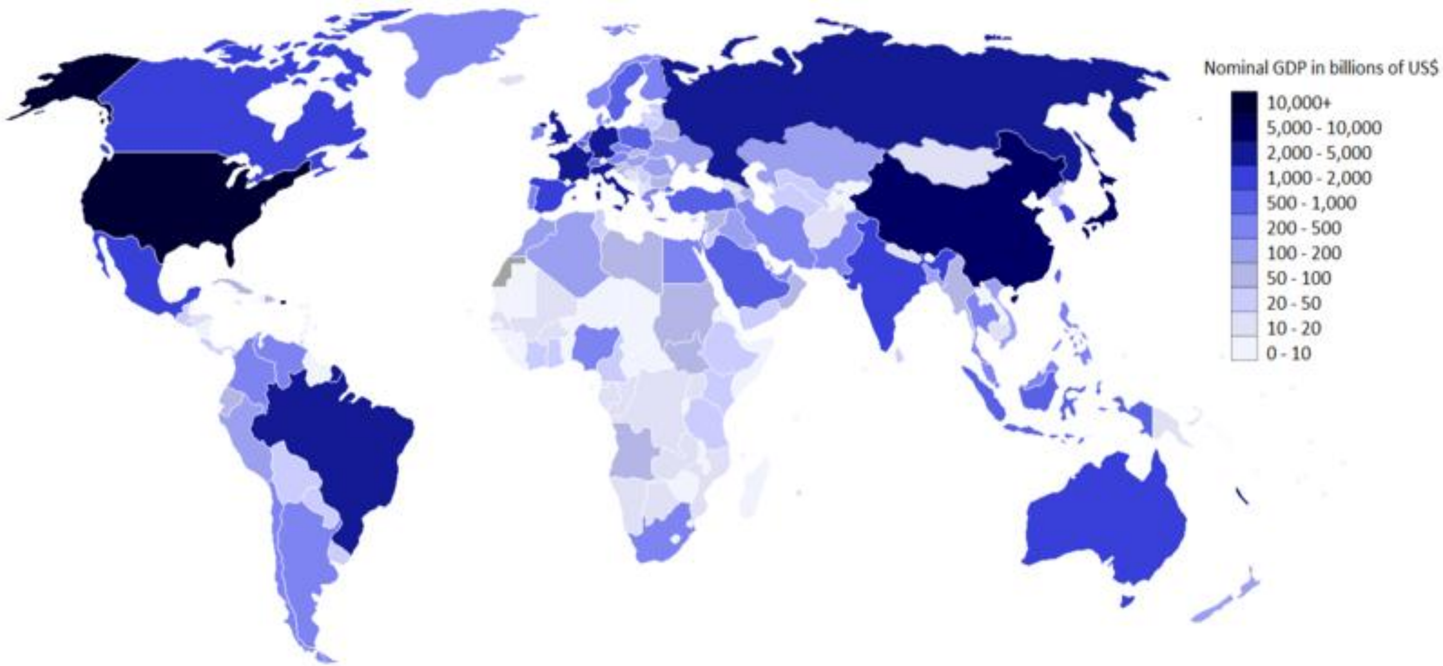
Lecture 6 Economic Growth I

- Accumulation of Capital
  - Supply and demand for goods
  - Growth in the Capital Stock and Steady State
  - How saving affects growth

- The Golden Rule Level of Capital
  - Comparing steady states
  - Finding the Golden Rule steady state
  - Transition to the Golden Rule steady state

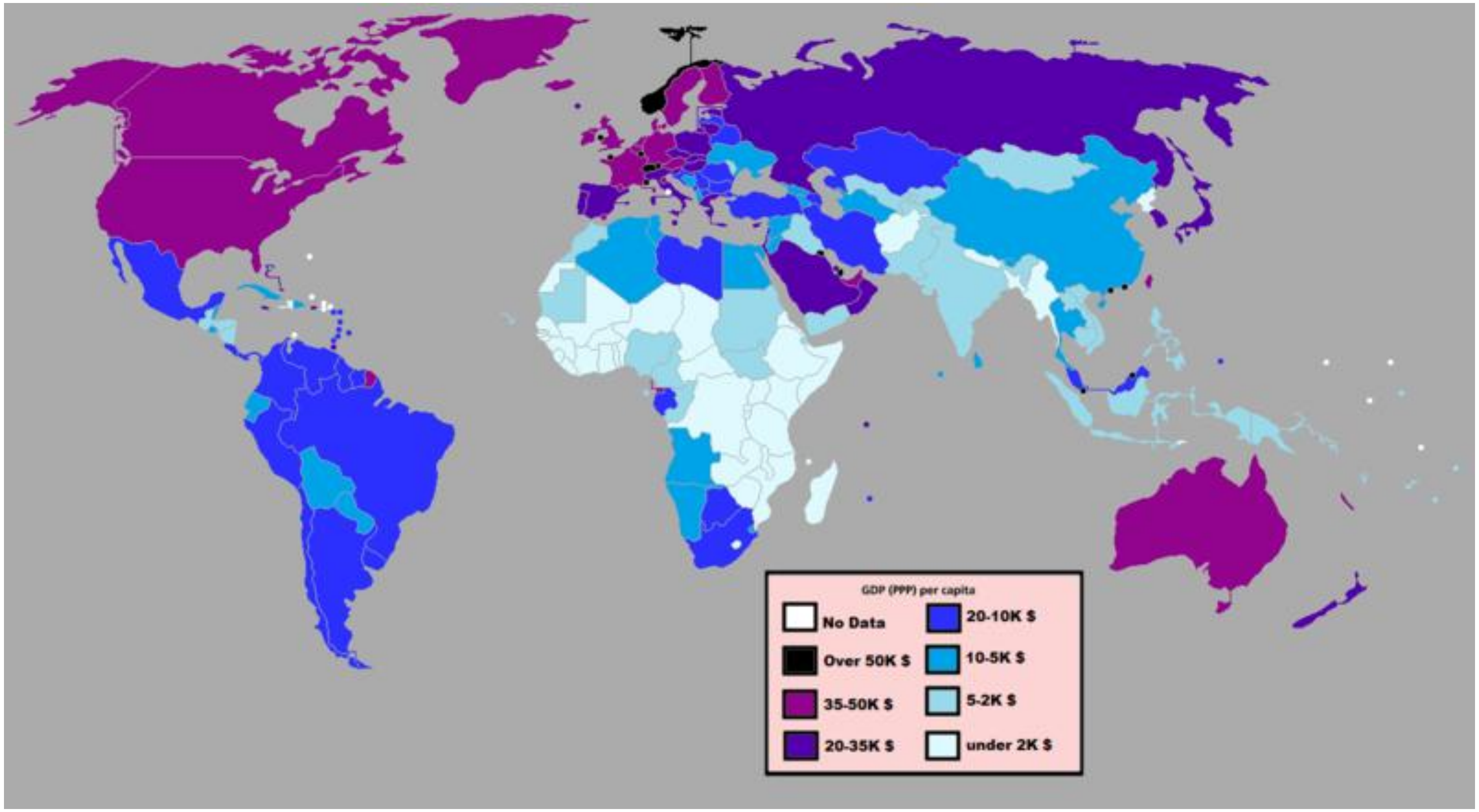
- Population Growth
  - The steady state with population growth
  - The effect of population growth

International Income Difference



We use GDP to measure **national income**. Data source: CIA, World Fact Book, 2012

International Living Standard Difference



We use **GDP per capita** to measure **living standard (生活水平)**. Data source: World Bank, 2011

The Solow Model

- Solow Model (索罗模型): a *dynamic(动态)* model of economic growth
  - growth in capital stock (资本积累)
  - growth in labor force (人口增长)
  - advances in technology (技术进步)

- We start by looking at **capital accumulation** (supply and demand in goods)

Robert Solow, Nobel Laureate (1987) and PhD advisor for another three Nobel Laureate:
Peter Diamond (2010), Joseph Stieglitz (2001) and George Akerlof (2001)

Production Function

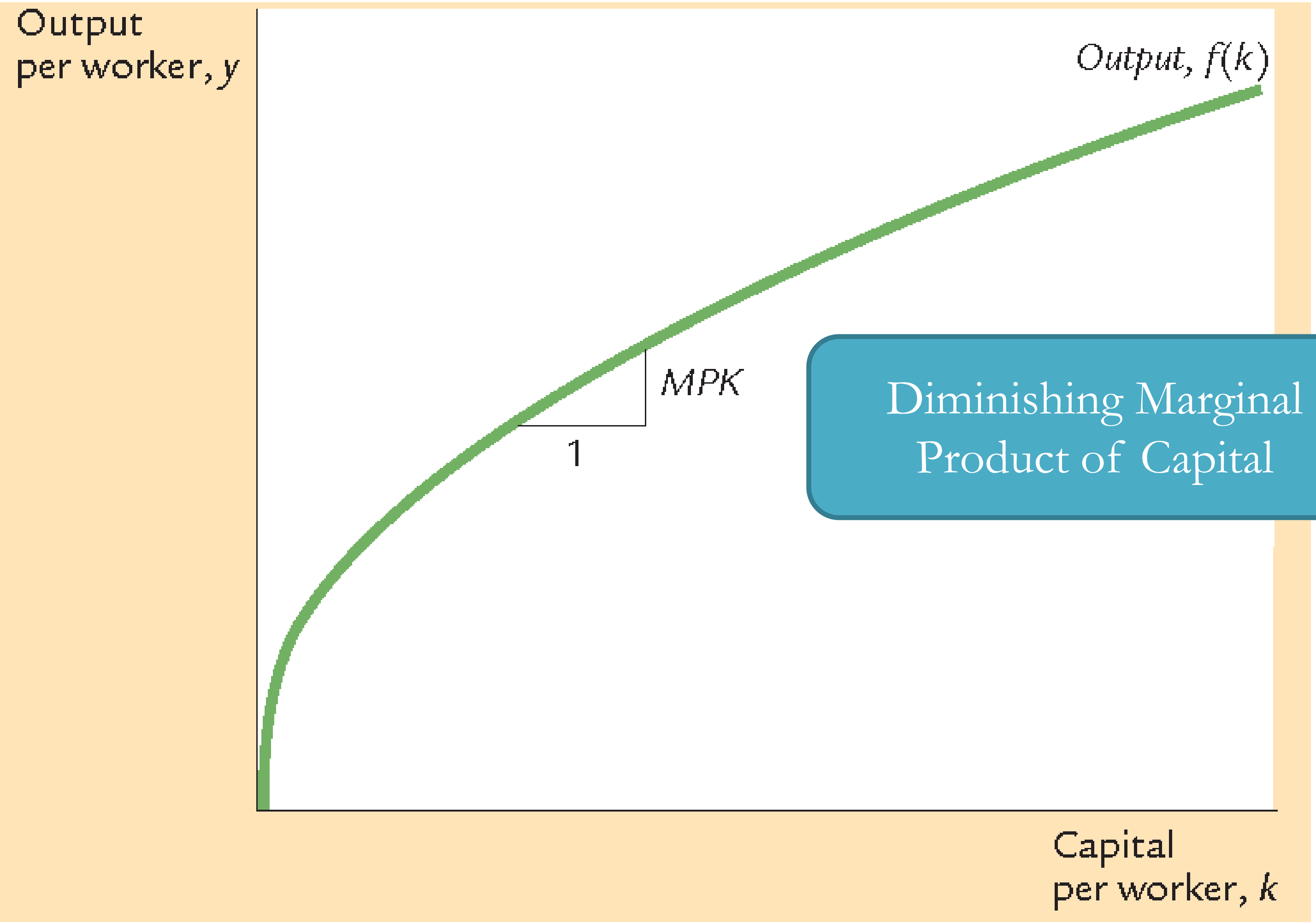
- In aggregate terms: $\boldsymbol{Y} = \boldsymbol{F}(\boldsymbol{K}, \boldsymbol{L})$
- Define: $\boldsymbol{y} = \boldsymbol{Y/L}$ = output per worker
  $\boldsymbol{k} = \boldsymbol{K/L}$ = capital per worker
- Assume **constant returns to scale** (规模报酬不变):
  $\boldsymbol{zY} = \boldsymbol{F}(\boldsymbol{zK}, \boldsymbol{zL})$ for any $\boldsymbol{z} > 0$
  - why we need this assumption?

The assumption of constant returns to scale ensures that the economic *size does not matter* for the properties of the production function, and therefore we can conveniently express the production function as an *univariate function*.

- Pick $\boldsymbol{z} = \boldsymbol{1/L}$. Then
  $\boldsymbol{Y/L} = \boldsymbol{F}(\boldsymbol{K/L}, \boldsymbol{1})$
  $\boldsymbol{y} = \boldsymbol{F}(\boldsymbol{k}, \boldsymbol{1})$
  $\boldsymbol{y} = \boldsymbol{f(k)}$ where $\boldsymbol{f(k)} = \boldsymbol{F}(\boldsymbol{k}, \boldsymbol{1})$
- The slope of the production function is the **marginal product of capital** (边际资本产出)

$$MPK=f(k+1)-f(k)$$

Production Function



Diminishing Marginal Product of Capital

Consumption Function

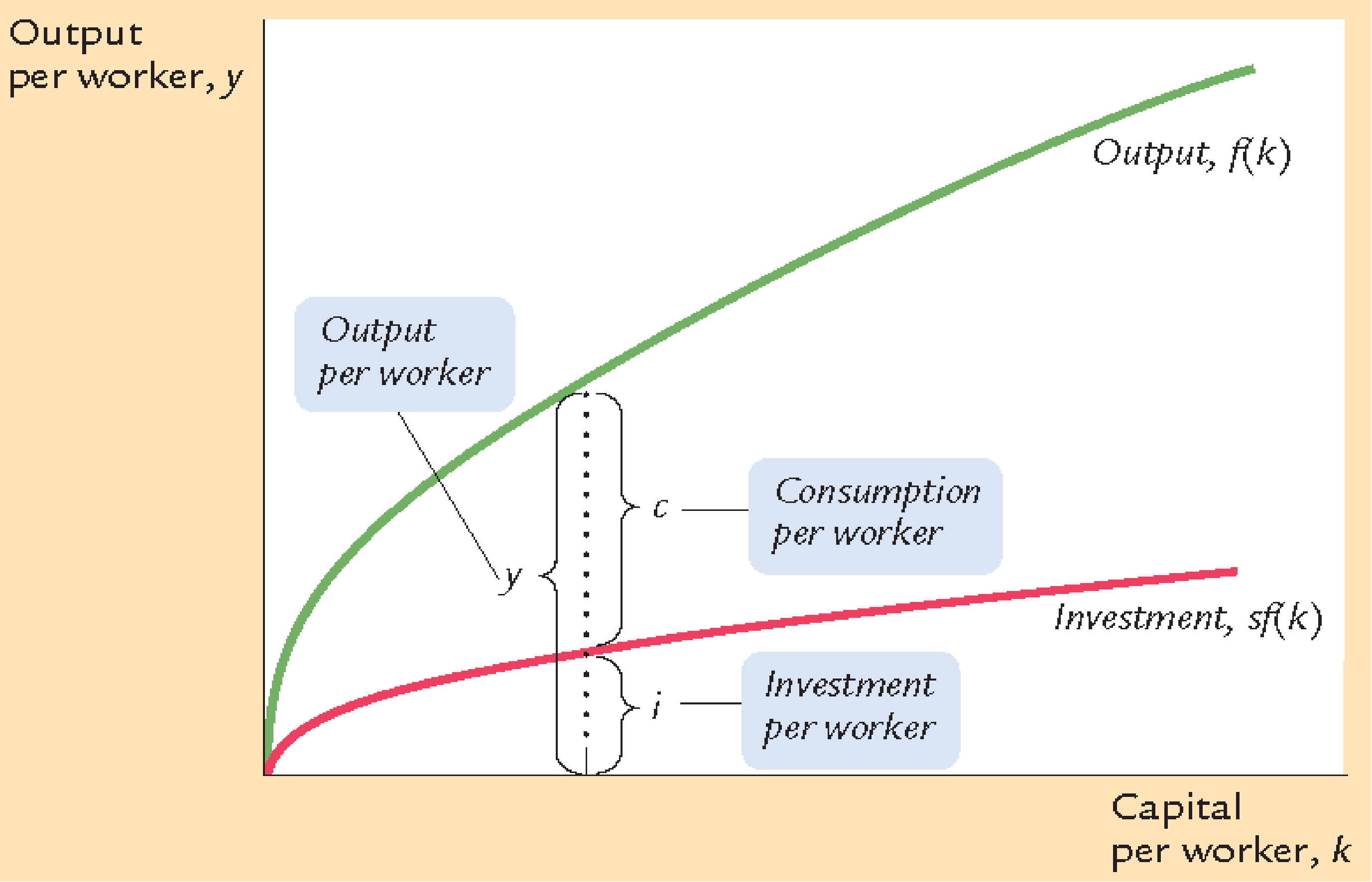
- $\boldsymbol{Y = C + I}$ (*no* $\boldsymbol{G}$ in the Solow model)
- In "per worker" terms: $\boldsymbol{y = c + i}$, where $\boldsymbol{c = C/L}$ and $\boldsymbol{i = I/L}$
- $\boldsymbol{s}$ = the saving rate,
  the fraction of income that is saved
  ($\boldsymbol{s}$ is an exogenous parameter)

The saving rate is actually more like a **policy variable**: both monetary and fiscal policies can affect the saving rate. Finding the optimal saving rate is precisely the primary objective of the Solow model.

- Consumption function: $\boldsymbol{c = (1-s)y}$ *(per worker)*
- **investment** per worker: **i = s y = sf(k)**

This equation relates the existing stock of capital *k* to the accumulation of new capital *i*.

Output, Consumption and Investment



Motion of the Capital

- **Depreciation** (折旧): wearing out of the old capital, making the stock of capital to fall
  - Depreciation rate (折旧率)： the fraction of capital that being wear out at each period

- Change in capital stock= investment –depreciation

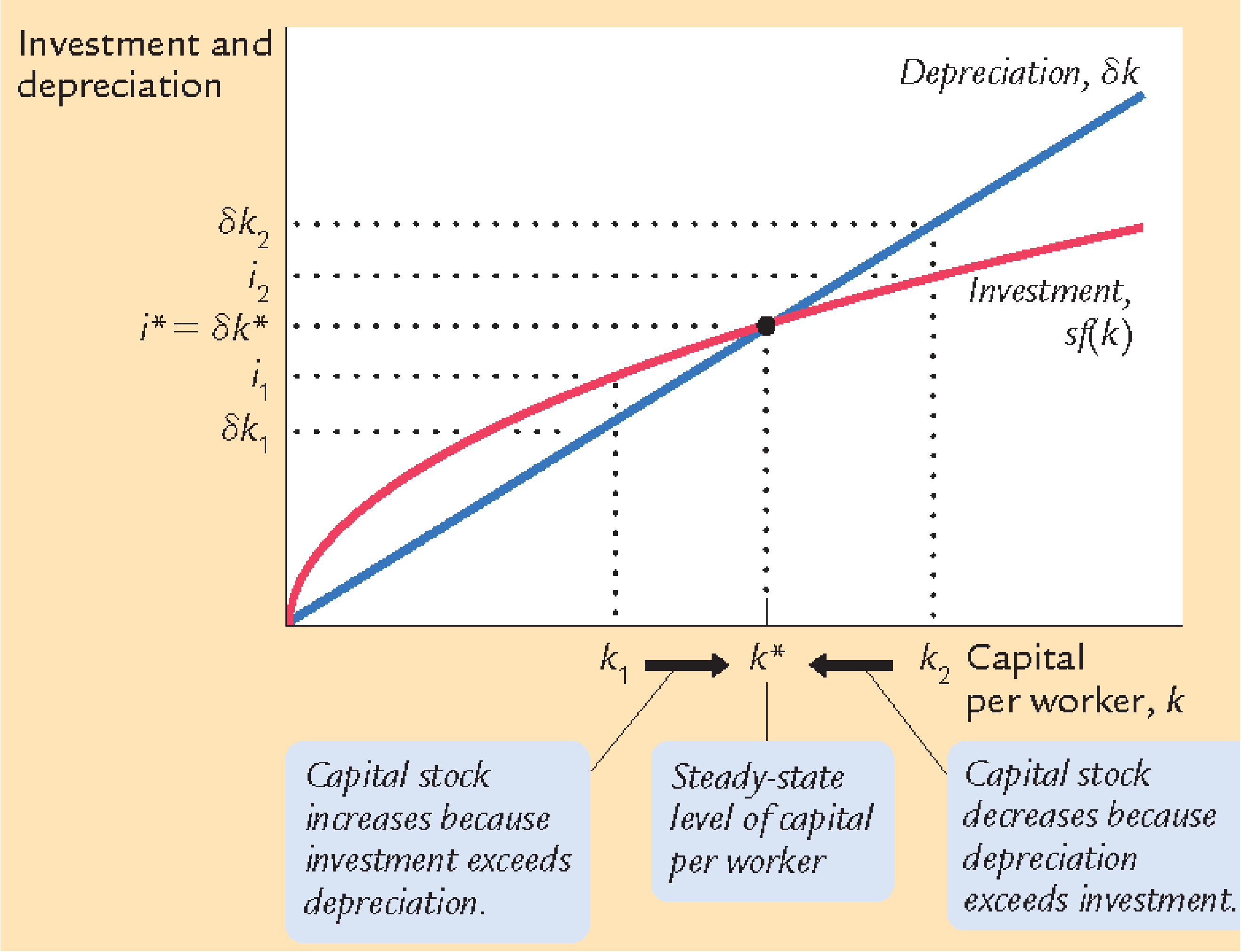
$$\Delta k = i - \delta k$$

Since $i = sf(k)$, this becomes:

$$\Delta k = sf(k) - \delta k$$

At the **steady state**, $k = k^*$, $\Delta k = 0$, and $sf(k^*) = \delta k^*$.

Investment, Depreciation and the Steady State



Why the investment curve and depreciation curve must intercept?

A Numerical Example

The aggregate production function is:

$$Y = F(K, L) = \sqrt{K \times L} = K^{1/2}L^{1/2}$$

The per-worker income is:

$$\frac{Y}{L} = \frac{K^{1/2}L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2} \quad \text{or} \quad y = f(k) = k^{1/2}$$

Assume:

- $s = 0.3$
- $\delta = 0.1$
- initial value of $k = 4.0$

A Numerical Example

Approaching the Steady State: A Numerical Example

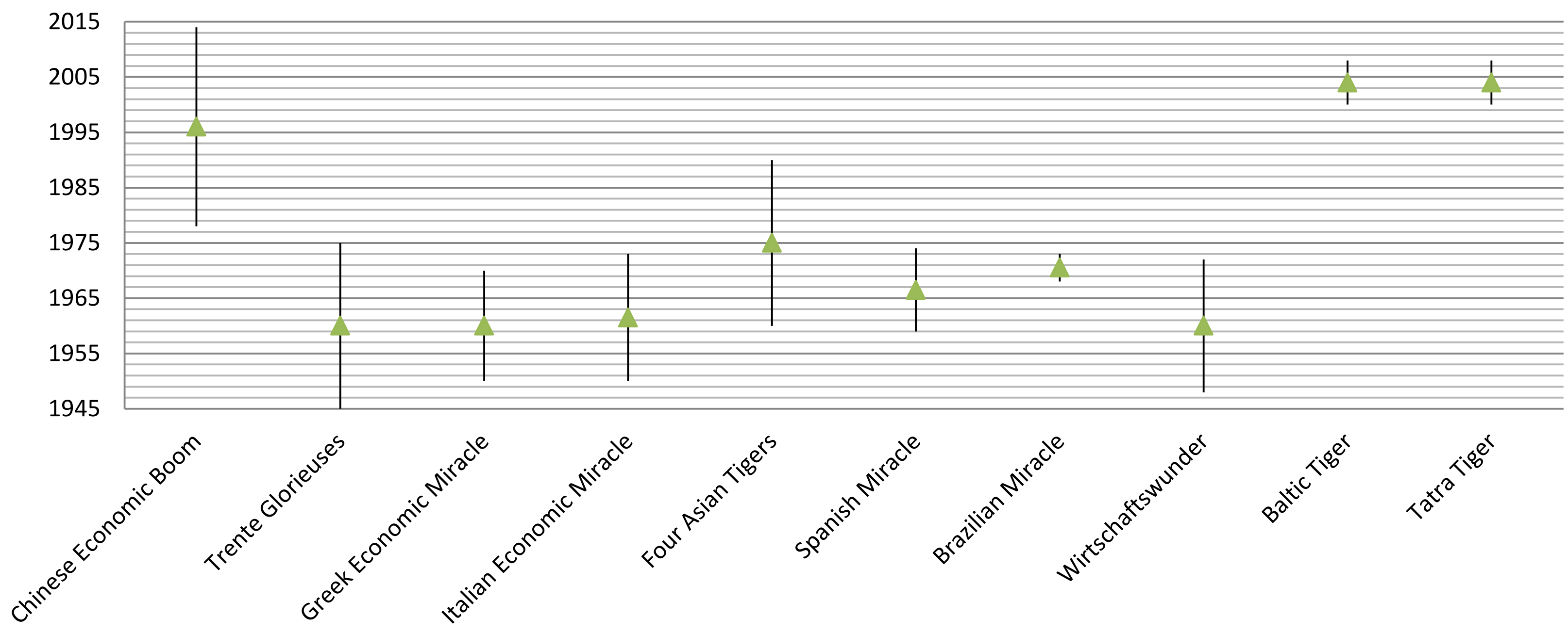
Assumptions: $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
.						
.						
.						
10	5.602	2.367	1.657	0.710	0.560	0.150
.						
.						
.						
25	7.321	2.706	1.894	0.812	0.732	0.080
.						
.						
.						
100	8.962	2.994	2.096	0.898	0.896	0.002
.						
.						
.						
∞	9.000	3.000	2.100	0.900	0.900	0.000

Alternatively we can use the equation:
$sf(k^*) = \delta k^*$
to calculate the ***steady state*** capital stock and the corresponding output, consumption, and investment.

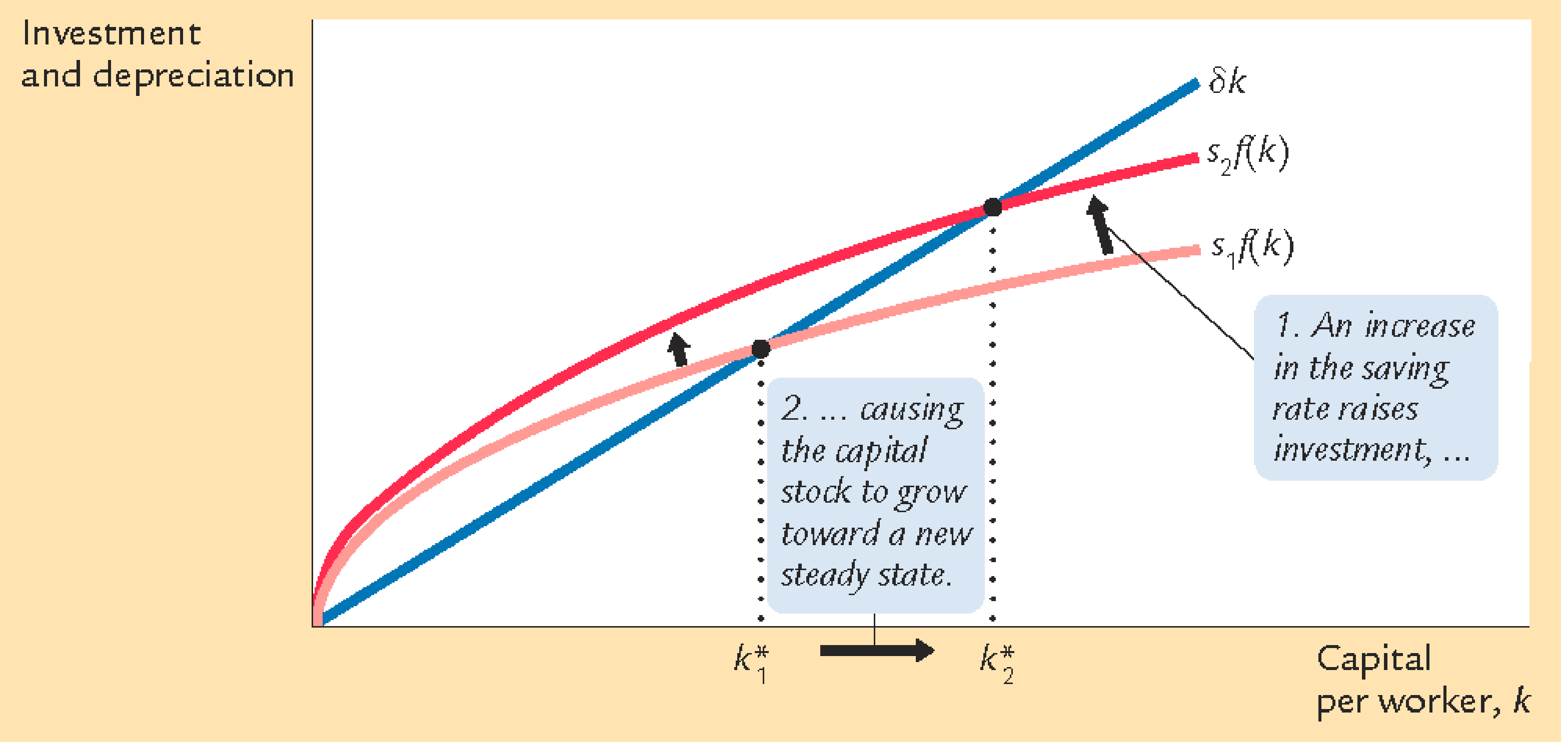
Economic Miracles

Economic miracle (经济奇迹) is an informal economic term commonly used to refer to a period of *dramatic economic development* that is entirely *unexpected* or *unexpectedly strong*.



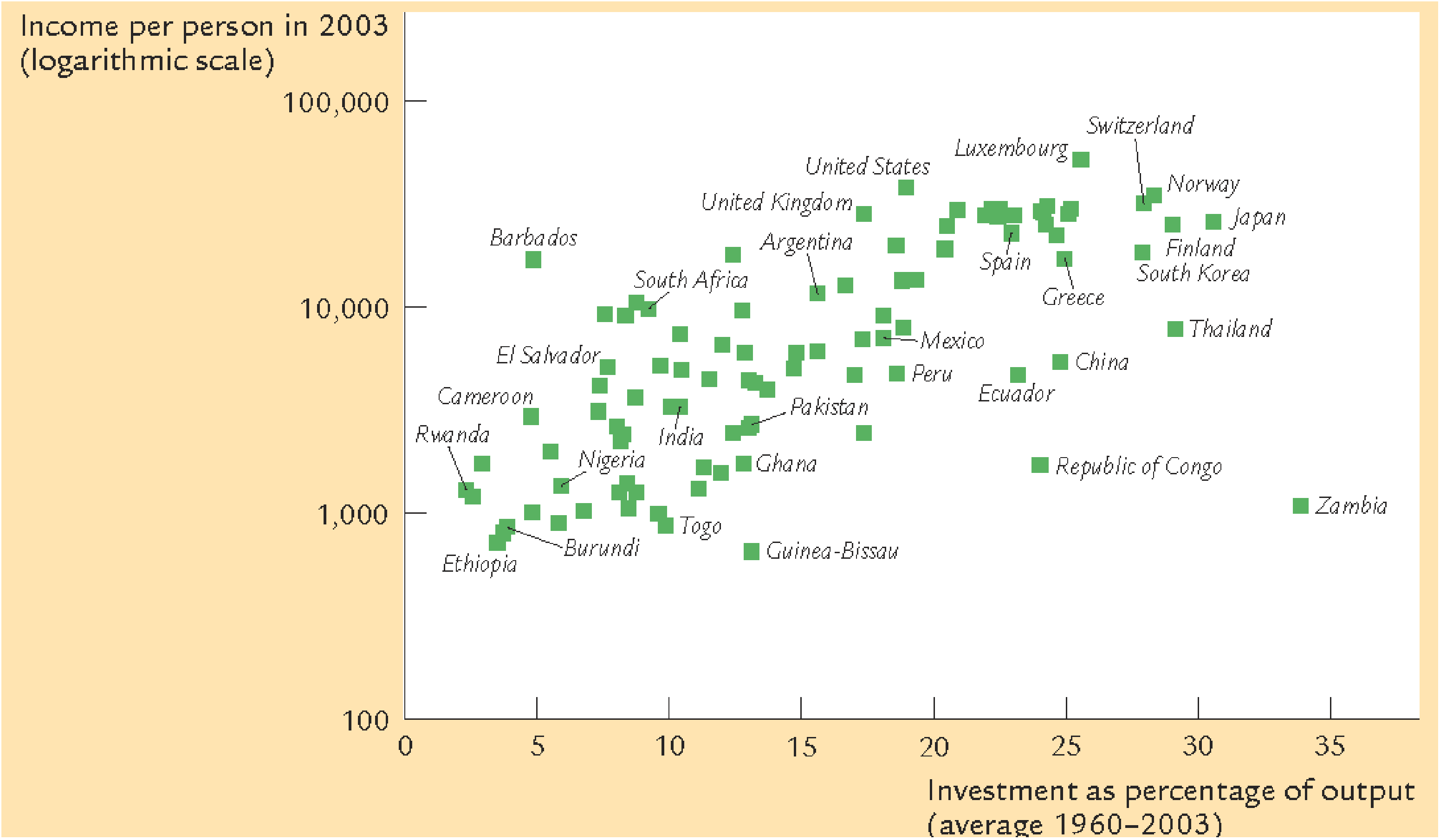
Saving and Economic Growth

- The post-war economic growth miracles in Japan and Germany are often attributed to their *low capital stock* (catch-up effect, 追赶效应) after WWII.
- Another important reason is the **high saving rate** (and hence high investment rate) in those two countries (and in many other countries such as China that have experienced exceptional rapid economic growth as well)



Saving and Income

The Solow Model predicts that countries with high saving rate have higher steady state capital stock per capita and hence higher income per capita. Is this really the case?



Saving and Economic Welfare

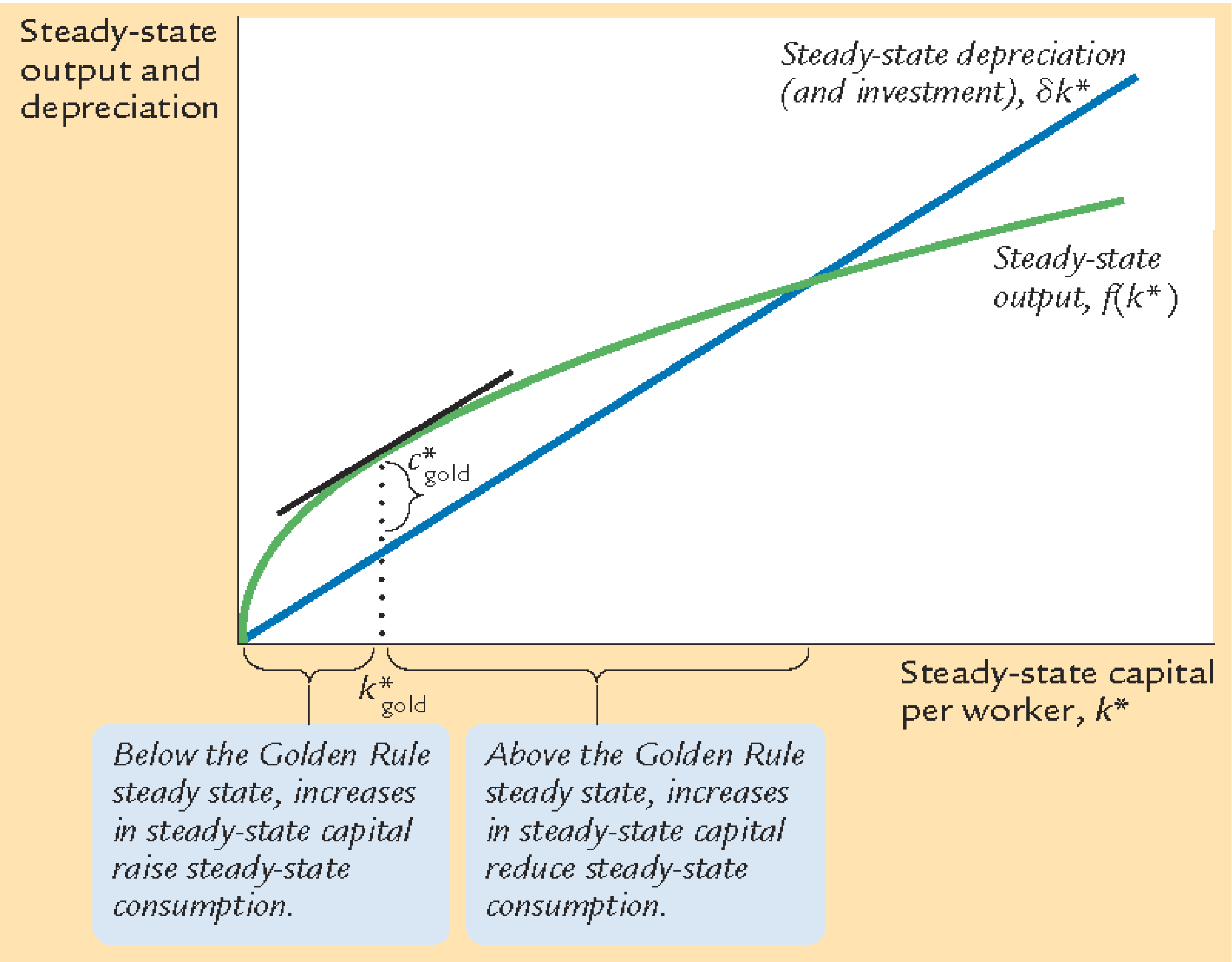
- Different values of s lead to different steady states. How do we know which is the "best" steady state?
- Economic well-being depends on consumption, so the "best" steady state has the highest possible value of consumption per person:

$$c^* = (1-s)\, f(k^*) = f(k^*) - \delta k^*$$

- An increase in s
  - ✓ leads to higher k^* and y^*, which may raise c^*
  - ✓ reduces consumption's share of income $(1-s)$, which may lower c^*
- So, how do we find the s and k^* that maximize c^*?

The steady-state value of k that maximizes consumption is called the **Golden Rule level of capital (资本的黄金律水平)**

The Golden Rule



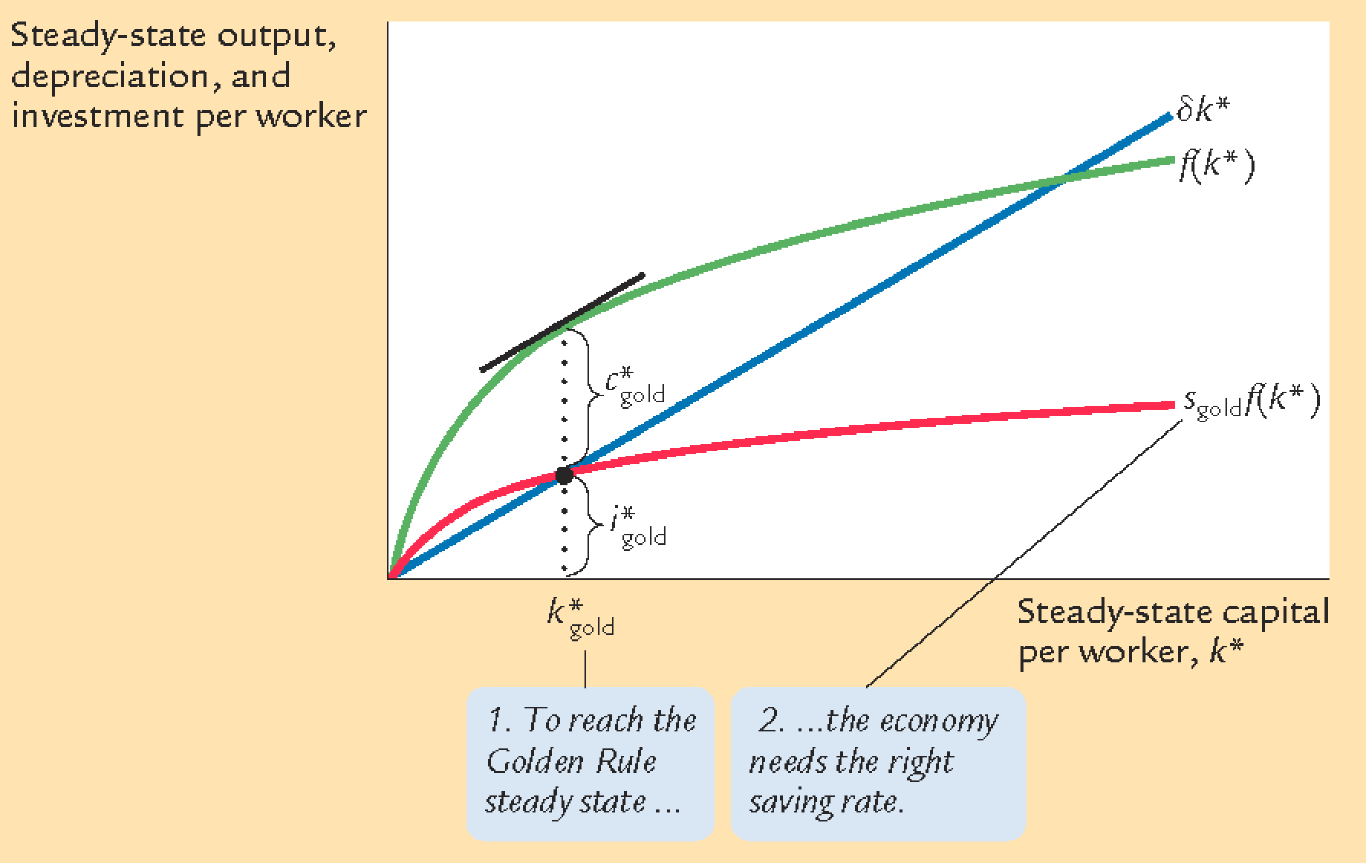
$c^* = f(k^*) - \delta k^*$

is biggest where the *slope of the production function* equals the *slope of the depreciation line*:

MPK= δ

The Golden Rule and Saving Rate

There is only one saving rate the produces the Golden Rule level of capita.



Finding the Golden Rule Steady State

Finding the Golden Rule Steady State: A Numerical Example

Assumptions: $y = \sqrt{k}$; $\delta = 0.1$

s	k^*	y^*	δk^*	c^*	MPK	$MPK - \delta$
0.0	0.0	0.0	0.0	0.0	∞	∞
0.1	1.0	1.0	0.1	0.9	0.500	0.400
0.2	4.0	2.0	0.4	1.6	0.250	0.150
0.3	9.0	3.0	0.9	2.1	0.167	0.067
0.4	16.0	4.0	1.6	2.4	0.125	0.025
0.5	**25.0**	**5.0**	**2.5**	**2.5**	**0.100**	**0.000**
0.6	36.0	6.0	3.6	2.4	0.083	–0.017
0.7	49.0	7.0	4.9	2.1	0.071	–0.029
0.8	64.0	8.0	6.4	1.6	0.062	–0.038
0.9	81.0	9.0	8.1	0.9	0.056	–0.044
1.0	100.0	10.0	10.0	0.0	0.050	–0.050

Transition to the Golden Rule Steady State

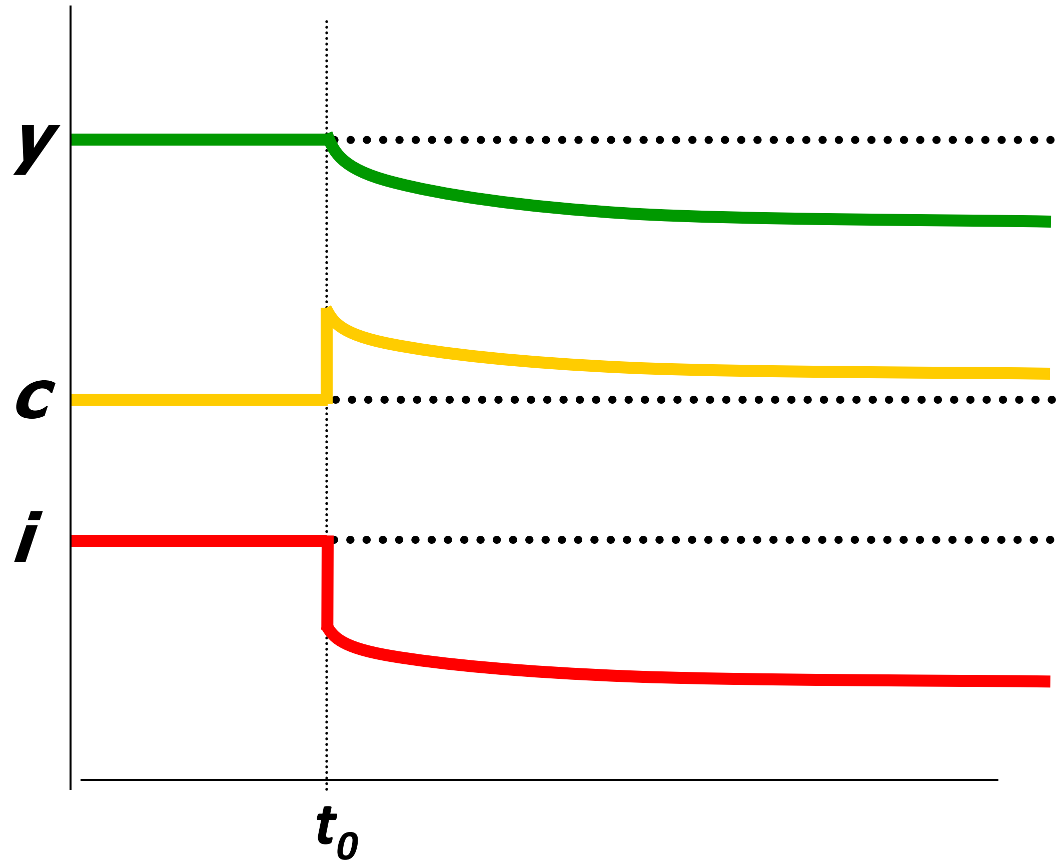
- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust ***s***.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with Too Much Capital

If $\boldsymbol{k}^* > \boldsymbol{k}^*_{gold}$

Then increasing c^* requires a fall in s.

In the transition to the Golden Rule, consumption is higher at all points in time.

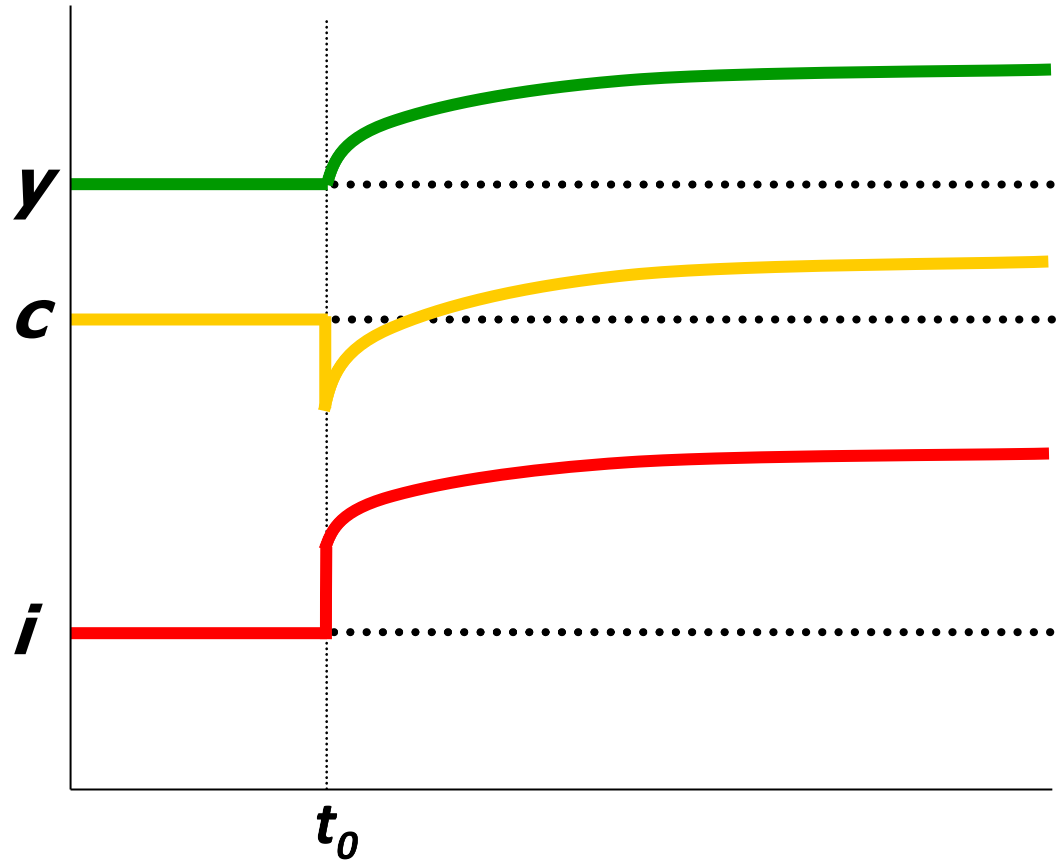


Starting with Insufficient Capital

If $k^* < k^*_{gold}$

then increasing c^* requires an increase in s.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



Population Growth

- The basic Solow model shows that capital accumulation, alone, **cannot** explain **sustained** economic growth (可持续的经济增长). Hence we need *population growth* and *technology advancement.* We start with population growth.

- Assume that the population--and labor force-- grow at rate $\boldsymbol{n}$. ($\boldsymbol{n}$ is exogenous)

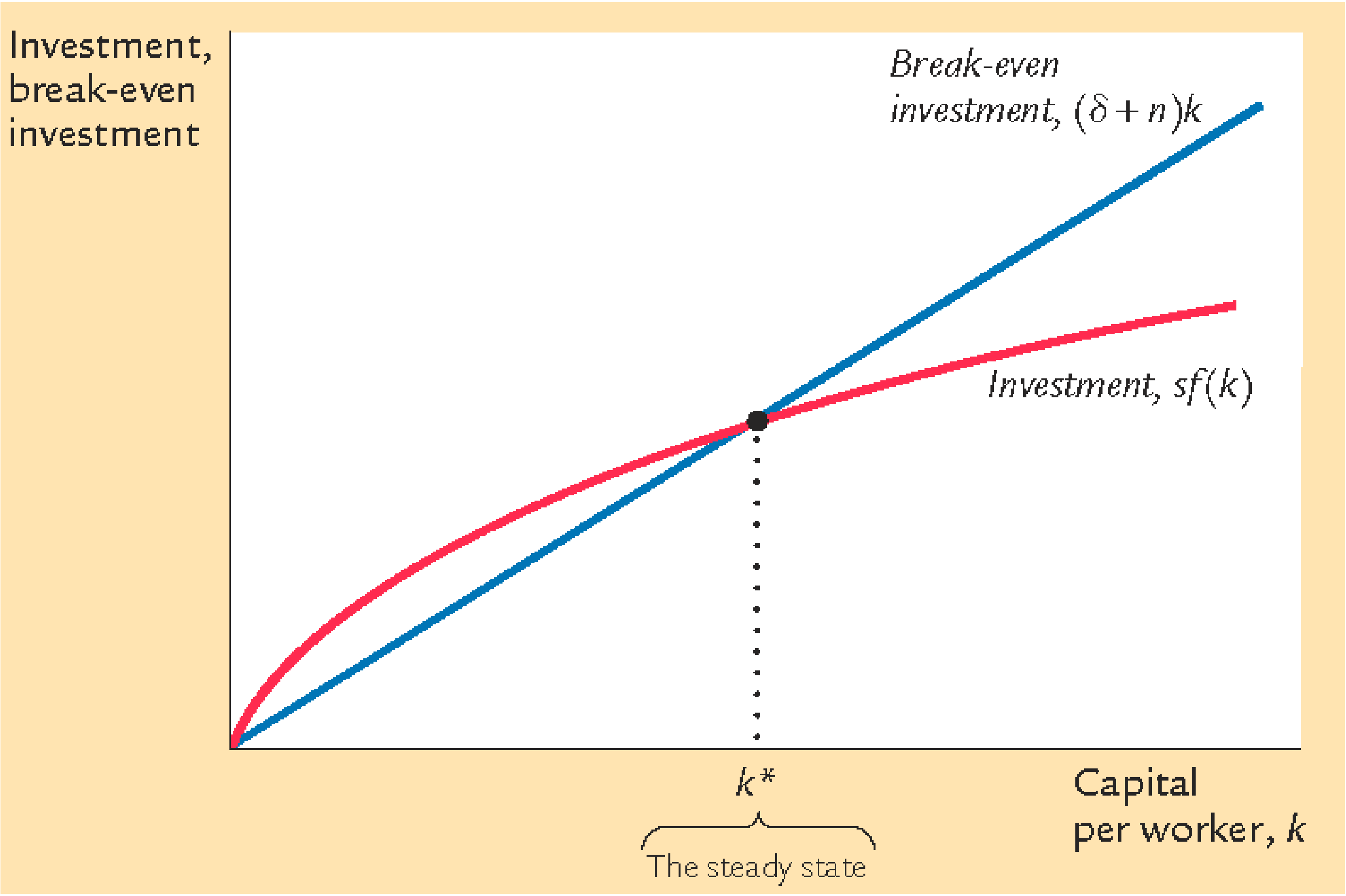
$$\frac{\Delta L}{L} = n$$

- $(\delta + n)k$ = **break-even investment**, the amount of investment necessary to keep $\boldsymbol{k}$ constant.

Break-even investment includes:

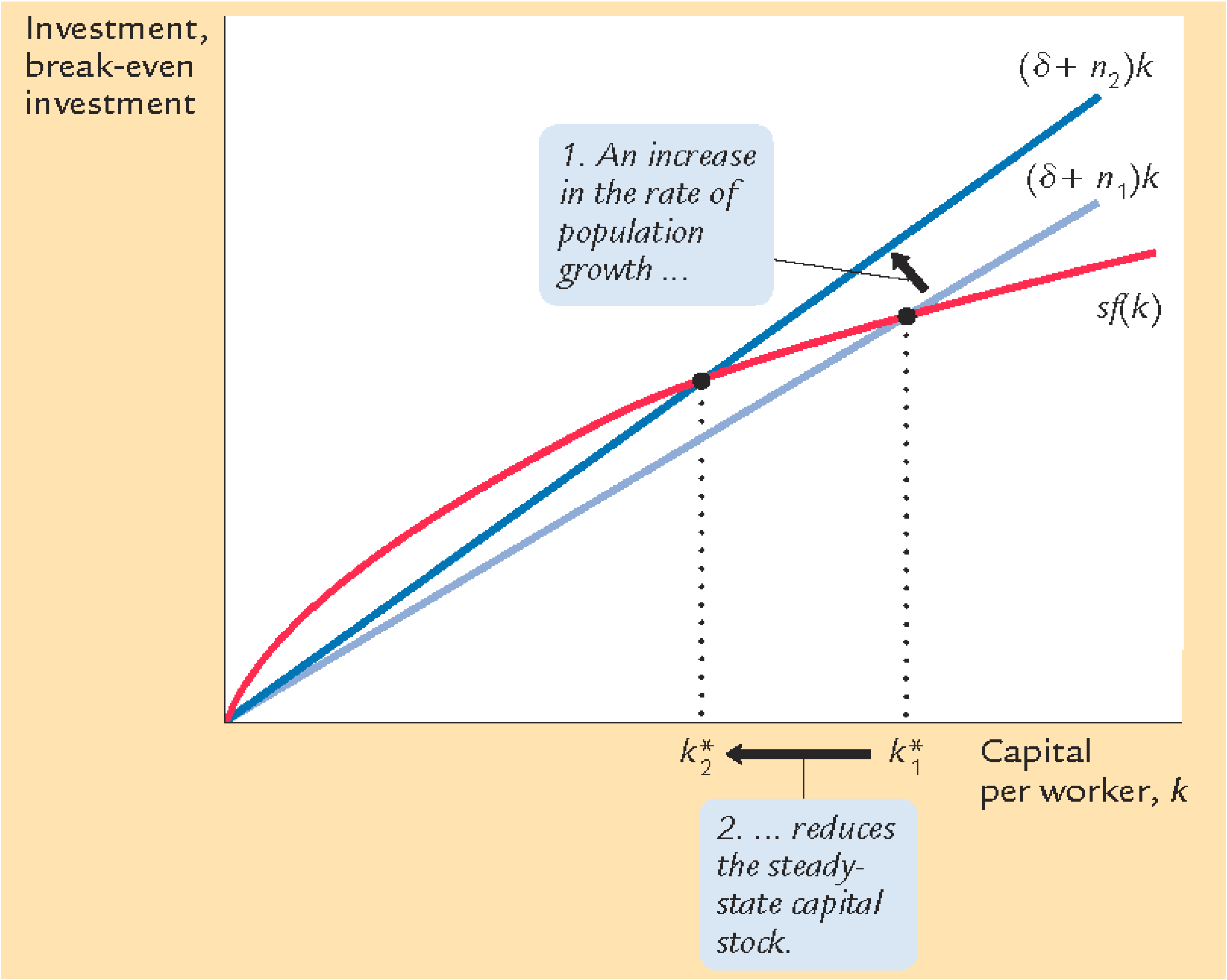
- δk to replace capital as it wears out
- $n k$ to equip new workers with capital
  (*otherwise,* $\boldsymbol{k}$ *would fall as the existing capital stock would be spread more thinly over a larger population of workers*)

Population Growth in the Solow Model



Effects of Population Growth

The Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run. Is this the case?



Population Growth and Income across Countries

Golden Rule with Population Growth

To find the Golden Rule capital stock, we again express c^* in terms of k^*:

$$c^* = y^* - i^*$$

$$= f(k^*) - (\delta + n)\, k^*$$

c^* is maximized when

$$\text{MPK} = \delta + n$$

or equivalently,

$$\text{MPK} - \delta = n$$